Machine-specified Ground Structures for Topology Optimization of Binary Trusses

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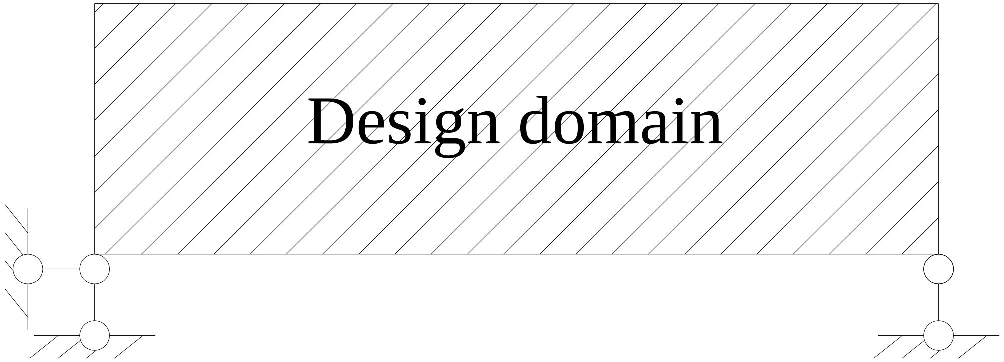
Background

Use AI to generate GSs?



Generate n nodes to make the design domain finite

***Number of members*:**

$n(n–1)/2$

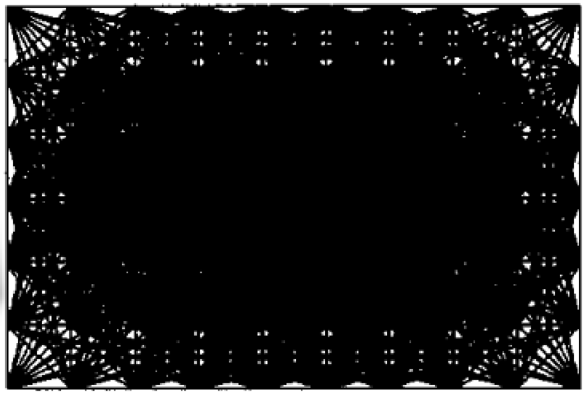
***Possible combinations*:**

$C^{n(n–1)/2}$

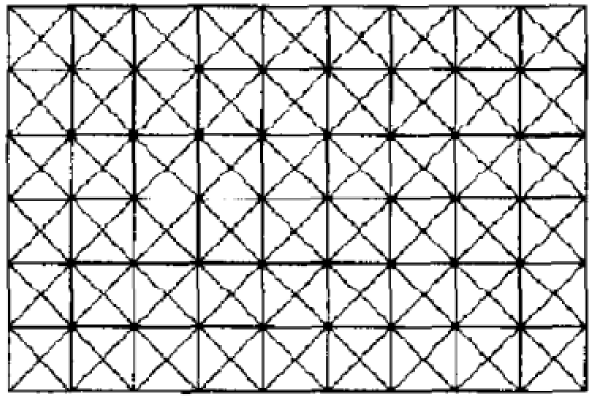
C: Number of sections

↓

High computational cost



Fully-connected ground structure (GS)



Human-specified GS

***Number of members*:**

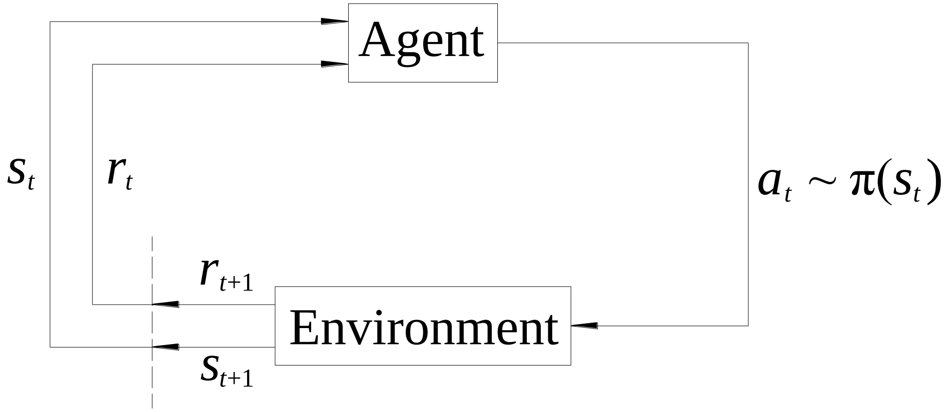
$2n - n_c < m < n(n–1)/2$

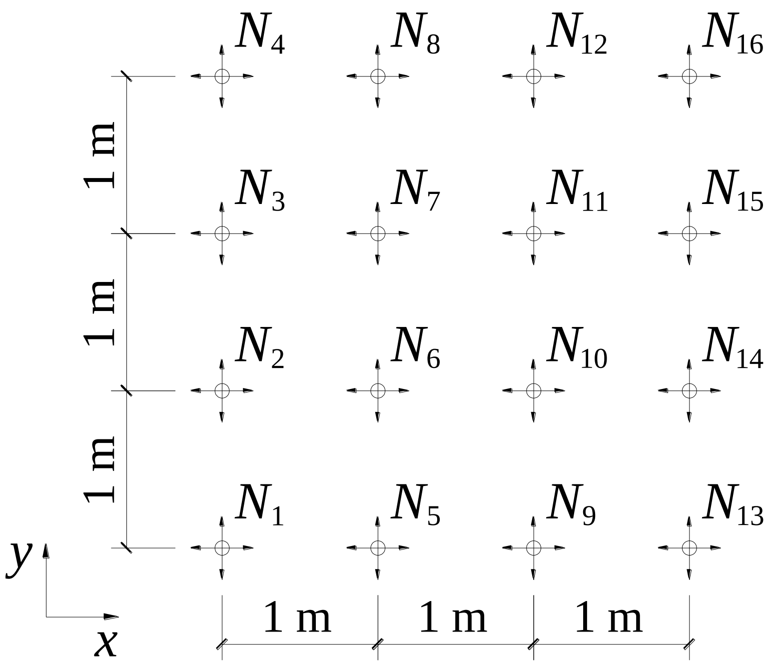
Kinematically stable

↓

Laborious to arrange

Reinforcement Learning (RL)





Task:

Generate a stable GS with fewest members

State:

Information on topology and nodal location

Action:

Select a node at a time
(2 selected nodes form a member)

Reward:

Positive for good actions
Negative for bad actions

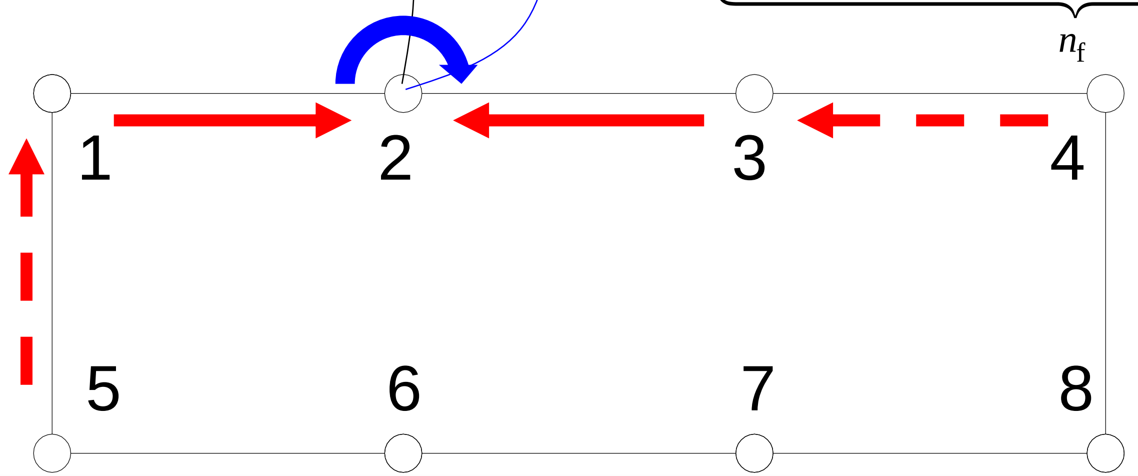
State description by Graph Embedding (GE)

nodal information vector

$$\mathbf{v}_i = \{x_i \quad y_i \quad \mathrm{Sel}_i\}$$

comprehensive information vector

$$\boldsymbol{\mu}_i = \underbrace{\{\mu_{i,1} \quad \mu_{i,2} \quad \mu_{i,3} \quad \cdots \quad \mu_{i,n_\mathrm{f}}\}}_{n_\mathrm{f}}$$



$$\hat{\mathbf{C}}_{8\times 8} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \end{array} \begin{array}{c} \begin{array}{cccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{array} \\ \begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{pmatrix} \end{array}$$

Neural network parameters (independent of *n*)

$$\boldsymbol{\mu}_i^{(T+1)} \leftarrow \mathrm{ReLU}\left(\boldsymbol{\theta}_1 \mathbf{v}_i + \boldsymbol{\theta}_3 \mathrm{ReLU}\left[\boldsymbol{\theta}_2\left(\hat{\mathbf{v}}\mathbf{C}_i^{(N)}\right)\right] + \boldsymbol{\theta}_4 \boldsymbol{\mu}_i^{(T)} + \boldsymbol{\theta}_6 \mathrm{ReLU}\left[\boldsymbol{\theta}_5\left(\hat{\boldsymbol{\mu}}^{(T)}\mathbf{C}_i^{(N)}\right)\right]\right)$$

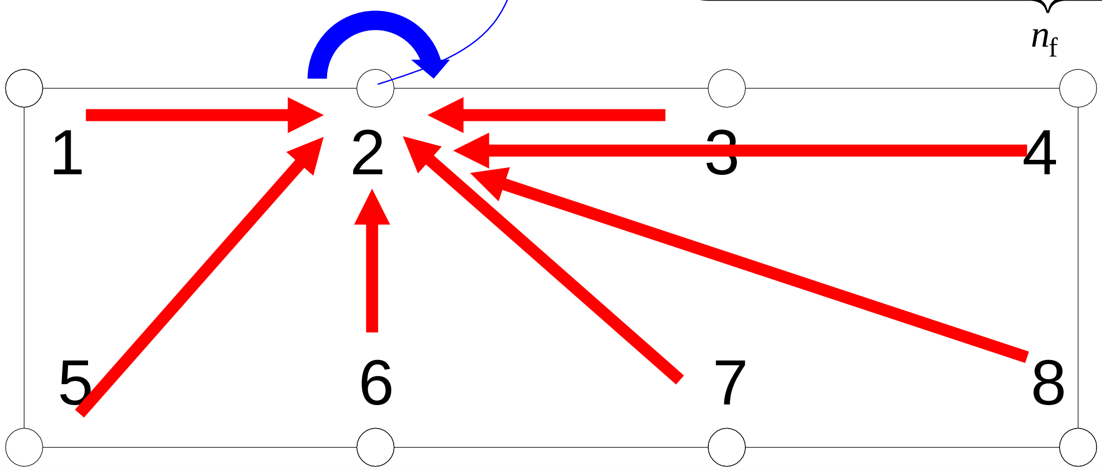
Information of the *i*th node

Information of adjacent nodes

Policy network

comprehensive information vector

$$\boldsymbol{\mu}_i = \{\underbrace{\mu_{i,1} \quad \mu_{i,2} \quad \mu_{i,3} \quad \cdots \quad \mu_{i,n_\mathrm{f}}}_{n_\mathrm{f}}\}$$



action probability

$$\boldsymbol{\pi} = \begin{Bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \\ p_6 \\ p_7 \\ p_8 \end{Bmatrix} = \begin{Bmatrix} 10\% \\ 15\% \\ 15\% \\ 10\% \\ 10\% \\ 15\% \\ 15\% \\ 10\% \end{Bmatrix}$$

Neural network parameters (independent of *n*)

$$Q_i = \mathrm{ReLU}\left[\boldsymbol{\theta}_9^{\mathrm{T}} \mathrm{ReLU}\left(\boldsymbol{\theta}_7 \boldsymbol{\mu}_i^{(T)}; \boldsymbol{\theta}_8 \sum_{i=1}^{n} \boldsymbol{\mu}_i^{(T)}\right)\right]$$

$$\boldsymbol{\pi} = \mathrm{Softmax}\left[\mathbf{S} \cdot \mathbf{Q}(\hat{\boldsymbol{\mu}})\right] = \frac{e^{\mathbf{S} \cdot \mathbf{Q}(\hat{\boldsymbol{\mu}}, i)}}{\sum_{i' \in \mathcal{A}} e^{\mathbf{S} \cdot \mathbf{Q}(\hat{\boldsymbol{\mu}}, i')}}$$

Information of the *i*th node

Information of all nodes

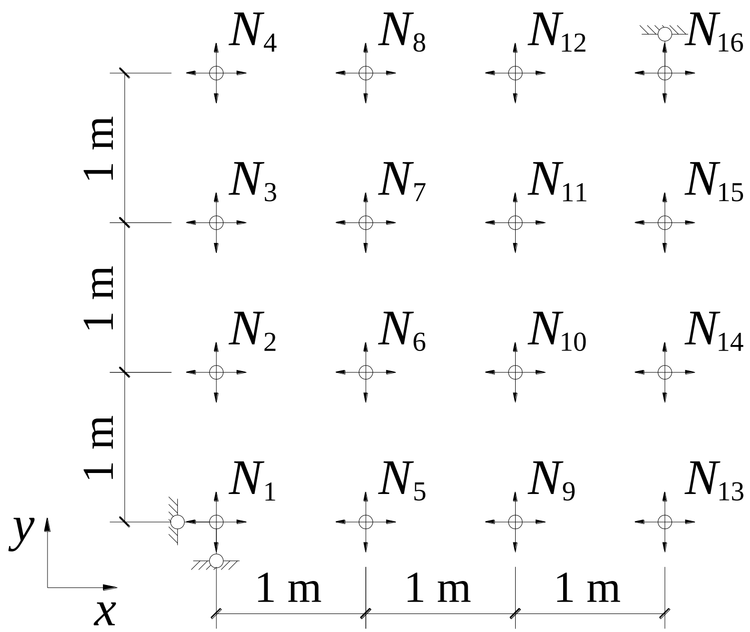
Learning using REINFORCE

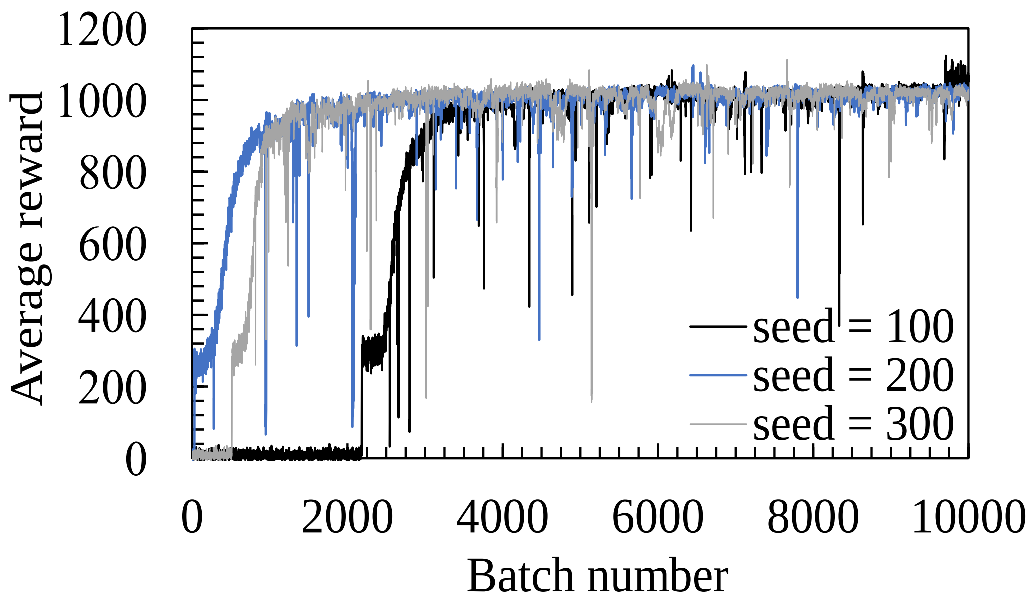
Estimated by Monte-Carlo sampling

Target: Maximize the total reward J

$$\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} + \alpha \nabla_{\theta} J(\boldsymbol{\theta}) \qquad \nabla_{\theta} J(\boldsymbol{\theta}) = \left(\frac{\partial J(\boldsymbol{\theta})}{\partial \theta_1}, \frac{\partial J(\boldsymbol{\theta})}{\partial \theta_2}, \ldots, \frac{\partial J(\boldsymbol{\theta})}{\partial \theta_{n_p}} \right)^{\mathrm{T}}$$

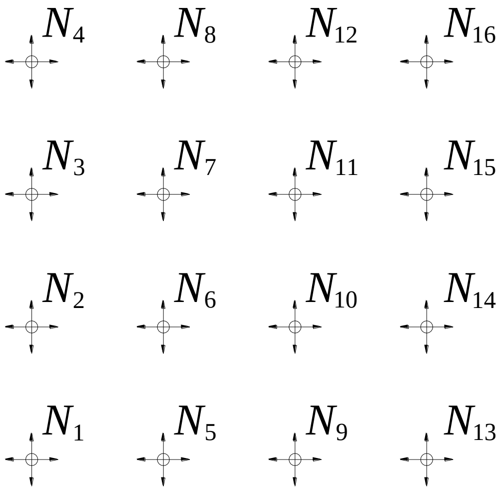
Effect: Probability of good actions ↑

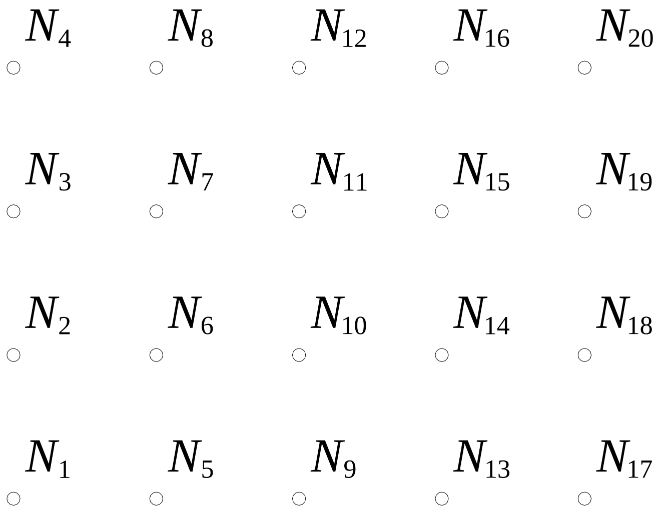




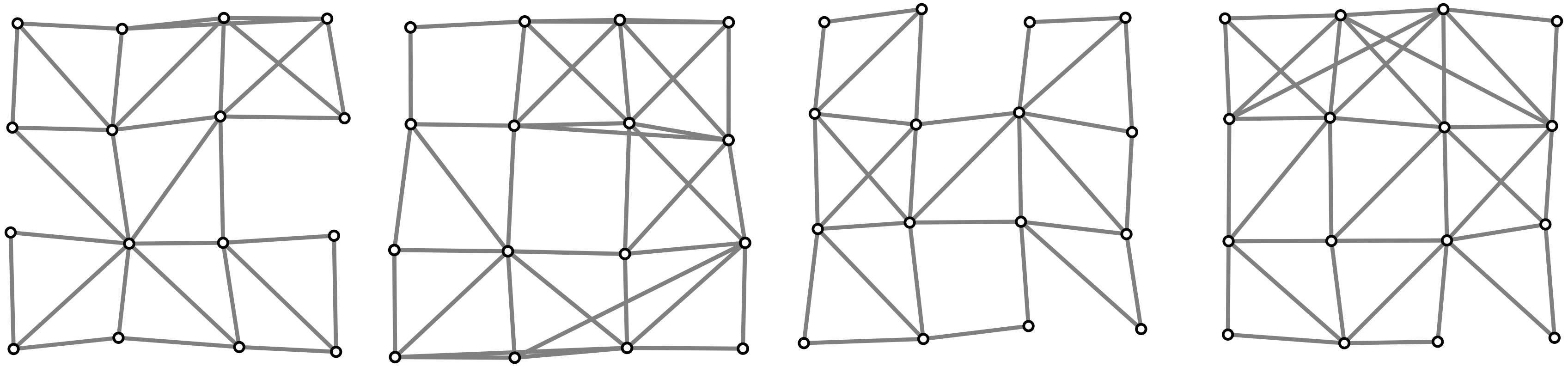
Testing of agent

Initial node-sets

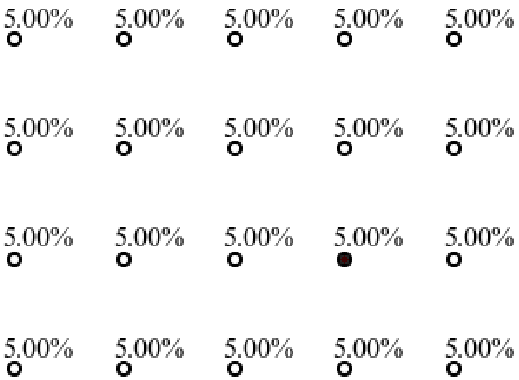


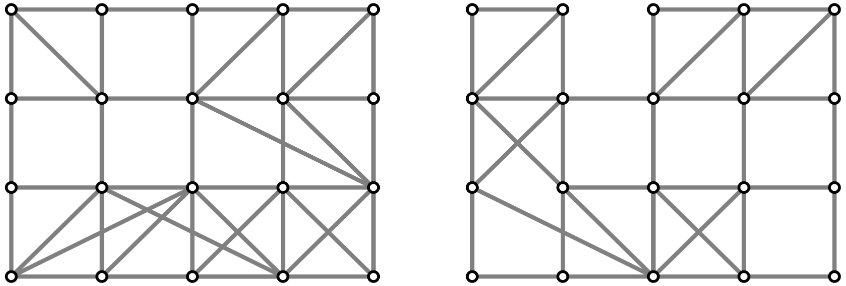


Generated trusses



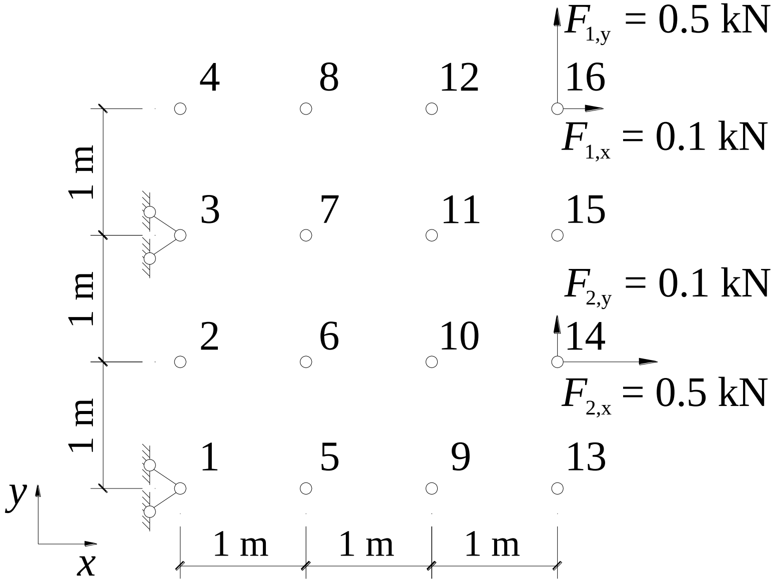
(node-set for training)





(different-sized node-set without re-training)

MGSs for topology optimization



Typical topology optimization problem with singular optimal solutions:

$$\begin{cases} \text{find} & \mathbf{A} \in \text{Section library} \\ \text{min.} & V(\mathbf{A}) \\ \text{s.t.} & \text{stress constraint} \rightarrow \max\limits_{i\in\Omega_m, j\in\{1,2,\cdots,n_L\}} \left(\dfrac{|\sigma_{i,j}(\mathbf{A})|}{\bar{\sigma}} \right) \le 1 \\ & \text{displacement constraint} \rightarrow \max\limits_{i\in\Omega_d, j\in\{1,2,\cdots,n_L\}} \left(\dfrac{|u_{i,j}(\mathbf{A})|}{\bar{u}} \right) \le 1 \\ & A_i \in \{\bar{A}\times 10^{-6}, \bar{A}\} \end{cases}$$

avoid unstable optimal solutions

Step 1: Generate a stable truss and randomly add to *m* members

Step 2: Remove the member with the lowest stress ratio until unstable

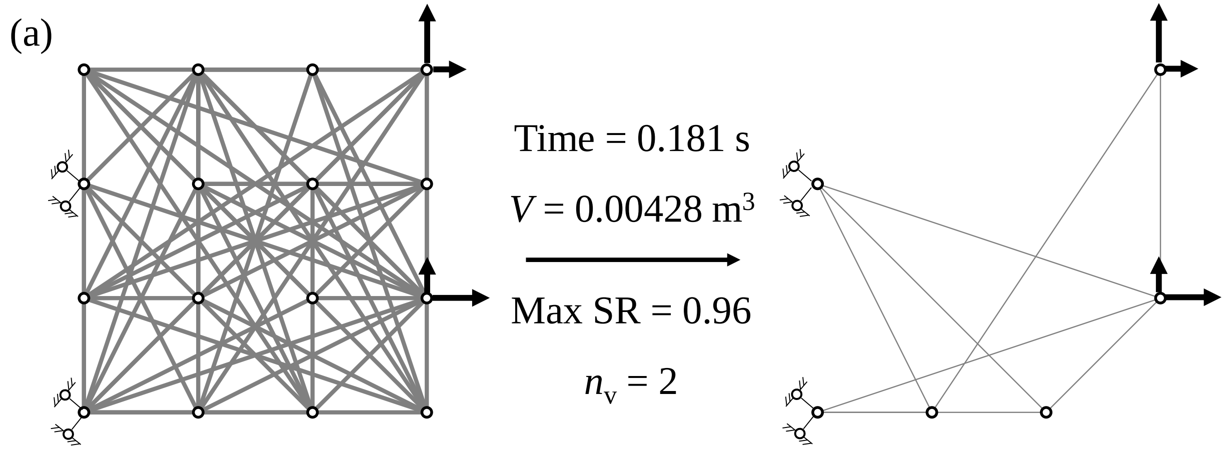
Step 3: Choose cross-sections according to fully-stressed design (FSD)

Numerical example

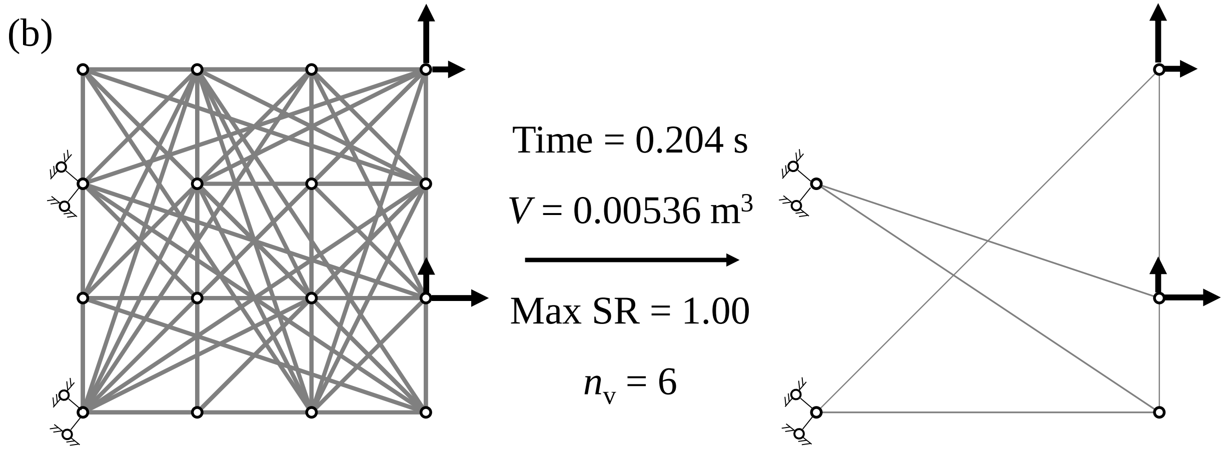
$n = 16 \quad m = 65$

$m_{min} = 2\times16-4 = 28$

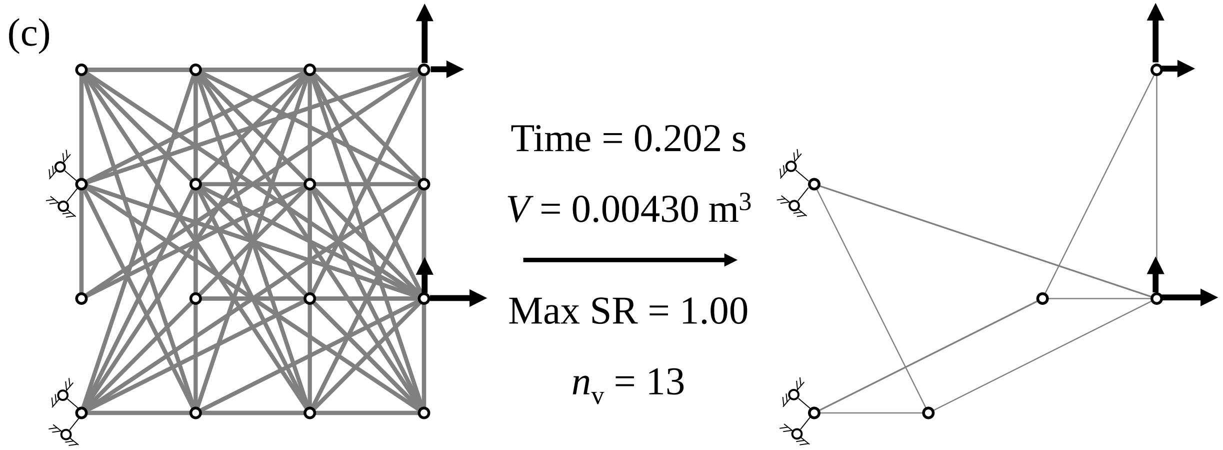
$m_{max} = 16\times(16-1)/2 = 90$



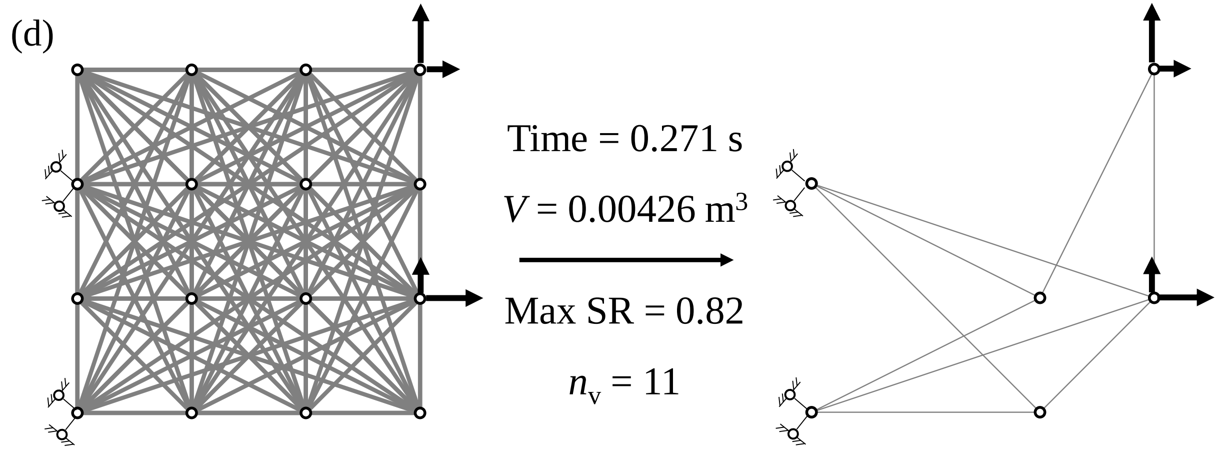
MGS1



MGS2



MGS3



Fully-connected GS

Contributions of this study

- An RL framework is proposed for MGSs
- A trained agent can generate a variety of MGSs
- Application of GE
  → different-sized problems without re-training
- MGSs lead to different optimal solutions
  → more possible to obtain the global optimum

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